Journal of Engineering Physics and Thermophysics, Vol. 77, No. 1, 2004

LOW-FREQUENCY EDDY CURRENTS AND DISSIPATIVE PROCESSES UNDER THE CONDITIONS OF MAGNETIC ABRASIVE TREATMENT

D. F. Ustinovich,[a] V. R. Sobol',[b] and O. N. Mazurenko[b]

UDC 621.923.4(087.2)

The questions of generation of eddy currents for a rotating cylindrical metallic part in magnetic abrasive treatment have been considered analytically. The power of dissipative electromagnetic forces and the effective braking torque have been calculated in the approximation of a normal skin effect on the basis of macroscopic field equations with the use of the model condition of axial symmetry for an electric field and the dispersion relation.

Formulation of the Problem. The occurrence of eddy currents in conducting materials is one of the main factors determining the phase shift between the magnetization and the external field and also the screening of the external field deep in the metal. The phase shift in turn leads to an irreversible dissipation of energy in a volume. At the same time, a magnetic field changing with time and the related flow induce electric currents which, according to the Lenz law, prevent the field in the material from changing and generate heat. Eddy currents have a screening effect, reducing the amplitude of the external field deep in the conducting material in accordance with processes of skinning, depending on the value and character of its magnetic permeability. On the other hand, the quasistatic magnetization of the materials is also accompanied by an irreversible ductile transformation of the field's energy, reflecting the fact that magnetic permeability can be a complex characteristic. In the low-frequency range of the spectrum, there occurs an extended maximum of the imaginary component of magnetic susceptibility, which assigns the scale of quasistatic magnetic viscosity and acts on frequency electromagnetic processes, leading to the generation of eddy currents [1].

In the course of magnetic abrasive treatment during rotation of the part in the stationary field of an electromagnet, the above effects of magnetic viscosity and of release of Joule heat by eddy currents are equivalent to the effective braking torque of counteracting forces. The data on the parasitic moment of magnetic forces are necessary to control the process of magnetic abrasive treatment, to develop equipment realizing the above method, and to select the technological parameters of the operation, which is usually carried out at a rotational frequency of the part of 50 to 300 Hz.

If the part being treated is made of a material whose magnetic permeability is real, the energy consumption by the effective braking will be determined only by eddy currents and their dissipation. In a complex type of magnetic permeability, an additional factor of the magnetic viscosity as a function of the imaginary component is present in an arbitrarily slow magnetization reversal of the part. Furthermore, under the conditions of nonstationary magnetization the complex character of magnetic permeability has an additional effect on the generation of eddy currents [2–5].

In the present work, we have considered the problem of the power of dissipative electromagnetic forces accompanied by magnetization and generation of currents in a cylindrically shaped part, rotating in the field of an electromagnet which is normal to its generatrix. The characteristic properties of irreversible dissipation of the energy of a magnetic field in the limiting case of normal skinning in the approximation of a local relationship between the current density and the electric field have been analyzed.

Analysis Procedure and Calculation Results. Use is made of macroscopic field equations which, in view of the low frequency of change of the magnetic field, employ the approximation of commensurability of the depth of the

---

[a]Physical-Technical Institute, National Academy of Sciences of Belarus, 10 Kuprevich Str., Akademgorodok, Minsk, 220141, Belarus; email: phti@ns.igs.ac.by; [b]Institute of Solid and Semoconductor Physics, National Academy of Sciences of Belarus, 17 P. Brovka Str., Minsk, 220072. Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 77, No. 1, pp. 163–166, January–February, 2004. Original article submitted August 27, 2002; revision submitted July 2, 2003.

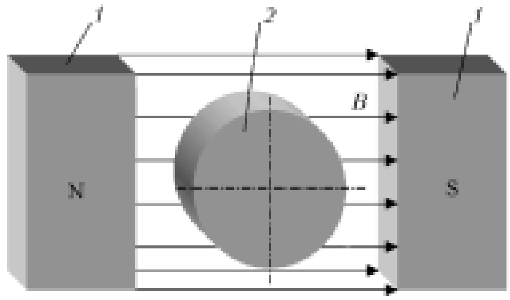

Fig. 1. Scheme of magnetic abrasive treatment: 1) pole pieces; 2) part.

skin layer with the radius of curvature the surface. In this case consideration is given to the problem where the rotating part is a solid cylinder which is located in a magnetic field changing with time. Figure 1 shows the most common scheme of magnetic abrasive treatment. During the rotation of the part in an external homogeneous field which is transverse with regard to its axis, at each point of the specimen's surface there are actually present two components of a magnetic field — azimuthal and radial ones, the sum of whose squares is equal to the square of the external field. During the rotation, these components change harmonically with time and transform into each other. The local dissipation of electromagnetic energy is in proportion to the sum of the squares of the amplitudes of the magnetic-field components, and the value of the released Joule heat is the same at each point of the surface in terms of the aggregate action of the radial and azimuthal components. This makes it possible to postulate, for simplification of calculation, a more symmetric character of the magnetic field in which case the field has only an azimuthal symmetry with an identical phase on the surface. The validity of this approach will be confirmed below by comparing the result obtained in the limit of low rotational frequencies and the scale of dissipation of energy during magnetization reversal in a quasistationary regime. During an infinitely slow change in the external field through a stationary part from the initial state to zero with subsequent magnetization in the opposite direction, demagnetization, and return to the initial state, the magnetic energy in the part's volume reaches the maximum two times. With account taken of the considerations expressed, we will analyze a model symmetric combination, using the formalism of a wave equation:

$$\nabla^2 E + k^2 E = 0 \,. \tag{1}$$

In cylindrical coordinates, with account taken of the postulated requirement of symmetry and a large length of the part $\partial E/\partial z = 0$ and $\partial E/\partial \vartheta = 0$ the wave equation is, in essence, a Bessel equation for the axial component of the vector of the electric field E in the volume

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + k^2 E = 0 \,. \tag{2}$$

The solution of this equation is a linear combination of the Bessel and Neumann cylinder functions, the coefficient of each of them being determined as an integration constant. Since the Neumann function grows infinitely for low values of r, we use a normally applied procedure to preserve the physical meaning and set the coefficient of the Neumann function to be zero. As a result, the solution contains only one constant A, which needs to be determined:

$$E = A I_0 (kr) \,, \tag{3}$$

where $I_0(kr)$ is a cylinder function of zero order of the third kind;

$$k^2 = \frac{i\omega\mu\mu_0}{\rho} \,. \tag{4}$$

Here $\mu = \mu_1 + i\mu_2$. To find the coefficient A we use a condition of relationship between the azimuthal component of the external magnetic field H^s and the axial component of the electric field E^s on the surface of the form

$$kE^s = \mu_0 \mu \omega H^s \tag{5}$$

The quantity of Joule energy dissipated in the volume of the specimen per a unit time can be determined by the method of summing up of the loss in the volume with the use of the relation

$$W' = \int_V \sigma E^2 dV \, . \tag{6}$$

We transform the power of the forces of dissipation of electromagnetic energy into an effective braking torque per unit length of the specimen, using the real part of expression (6), reduced to one period of rotation:

$$M = \frac{\mu}{i\mu_0} B^2 \pi R^2 \left[1 + \frac{I_1^2(kR)}{I_0^2(kR)} \right]. \tag{7}$$

The factor before the square bracket in dimension is the value of magnetic energy in the volume of a part of unit length. The presence of an imaginary unit in the denominator means that to determine the braking torque, it is necessary to calculate the imaginary part of the expression made up of the complex magnetic permeability and the sum in the square brackets, which is a function of the complex argument. For infinitely low frequencies, at which the contribution from electromagnetic induction disappears, the expression for an effective braking torque as a real part of the energy of dissipation transforms into a relation of quasistatic magnetization reversal with a viscous contribution to an irreversible component of the energy of a magnetic field of the form

$$M = \frac{\mu_2}{\mu_0} B^2 \pi R^2 \, . \tag{8}$$

If we consider the process of quasistatic magnetization reversal in a homogeneous magnetic field, in two acts of change of the field the magnetic energy will change by the value

$$W = \frac{4\mu\pi R^2}{(1+\mu)^2} \frac{B^2}{\mu_0} \, . \tag{9}$$

An analysis of expression (9) shows that its imaginary part converges to (8) in value for values of the real component of the magnetic susceptibility of the order of unity. For values of the real component of the magnetic permeability higher than unity the effective braking torque corresponds to the dependence

$$M \cong \frac{4B^2 \pi R^2}{\mu_0} \frac{\mu_2}{\mu_1^2 + \mu_2^2} \, . \tag{10}$$

With account taken of what has been said above, we can state that the relation (7) obtained is in a fairly good agreement with the results of static consideration of magnetization and viscosity at least for low-magnetic materials.

The analysis shows that the effective braking torque varies over wide limits as a function of the parameter kR. The results of calculation of the effective torque which are related to the magnetic viscosity determined by both the eddy nature and the static magnetic permeability are presented in Figs. 2 and 3 as a family of curves. In these figures, the independent variable is the radius of a cylindrical part and the angular velocity, whereas the imaginary component of the magnetic permeability is a postulated parameter which is present in calculations only for magnetic material. The real component of the magnetic permeability has been selected from the range characteristic of nonmagnetic and low-magnetic materials.

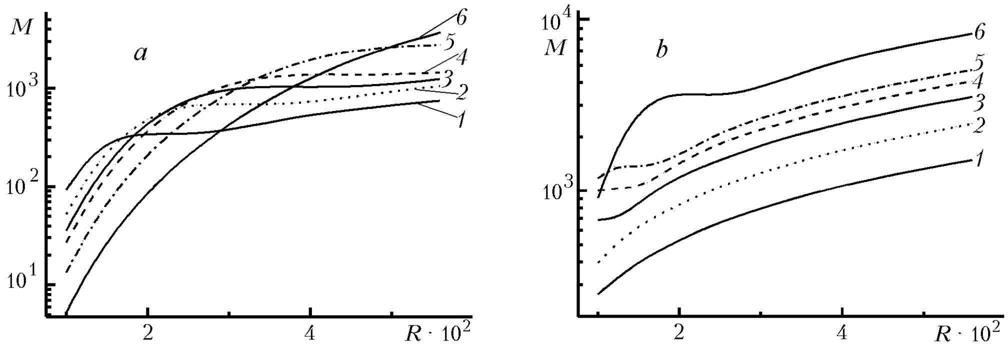

Fig. 2. Reduced moment of braking forces vs. radius of the part for different values of the resistance: 1) $\rho = 0.25 \cdot 10^{-7}$; 2) 0.5; 3) 0.75; 4) 1; 5) 2; 6) 5 Ω·m. $\omega = 350$ rad/sec. $\mu = 1$ (a); $\mu = 10 - i0.1$ (b). *M*, N; *R*, m.

The reduced torque as a function of the radius of the part shows a growth with an insignificant local extremum (Fig. 2). It is characteristic that in the region of low values of the radius where the thickness of the skin effect is comparable to the diameter, the nonmagnetic material with a lower specific resistance shows a larger scale of dissipative effects (Fig. 2a). This is caused by the higher value of the eddy currents in the axial nearly homogeneous field of the same amplitude for all values of the resistance of the material at the assigned rotational frequency. As the radius of the part increases, the situation in the distribution of braking forces becomes the opposite. For the part with a low resistance the braking forces are the smallest. In all probability, in this case we have the factor of electromagnetic screening where the energy release is localized only in the vicinity of the surface for a skin-layer thickness smaller than the part radius.

For the magnetic material the character of change in the reduced torque with increase in the radius of the part is more complex (Fig. 2b) and the intersection of characteristics is nearly absent. The large scale of the real part of the magnetic permeability is equivalent in action to conductivity and decreases the thickness of the skin layer, i.e., the degree of screening of the part's volume, which qualitatively corresponds in behavior to the curves in the right-hand part of Fig. 2a. The imaginary component additionally shifts the curves along the ordinate axis toward the region of higher values of the braking torque.

A general analysis of relation (7) shows that the effective braking torque as a function of the specific resistance is a nonmonotone characteristic. The nonmonotone nature is also characteristic of the frequency dependence of the reduced torque of a part of nonmagnetic material (Fig. 3a). For a wide range of values of the permeabilities of magnetic materials one should, probably, expect extrema on the characteristic of the reduced torque which are shifted toward the range of higher values of the frequency (Fig. 3b).

We note that for the assigned values of the radius and the rotational frequency the region of high specific resistance is characterized by the low values of the parameter kR ($kR \ll 1$) for which the value of the torque is determined by the volume effect of dissipation of the magnetic-field energy. With decrease in the wave vector in the region of low frequencies the level of dissipation decreases. The other limiting case is characterized by higher values of the parameter kR ($kR \gg 1$), as the values of the specific resistance decrease. In this limit, the skin-layer thickness decreases to such an extent with increase in the frequency that the total energy in the skin-layer volume is a decreasing function, which is consistent with the general notions of limiting transition. In other words, for a material possessing an infinitely high conductivity the external electromagnetic field cannot, in principle, penetrate into the volume and the eddy currents, put very simply, are not involved in energy dissipation.

The maximum on the dependence of the effective torque as a function of the specific resistance points to the transition of one mechanism of asymptotic behavior to another. It is characteristic that the behavior is qualitatively similar for different values of the initial magnetic permeability of the material. This points to the primary importance of nonstationary processes in energy dissipation.

In the case of a particular material, the braking torque will be determined by the magnetic permeability, the rotational frequency, the specific resistance, and the radius. To select the optimum regime of treatment and to decrease the braking torque one must ensure values of the wave vector and the technological parameters preventing the maxi-

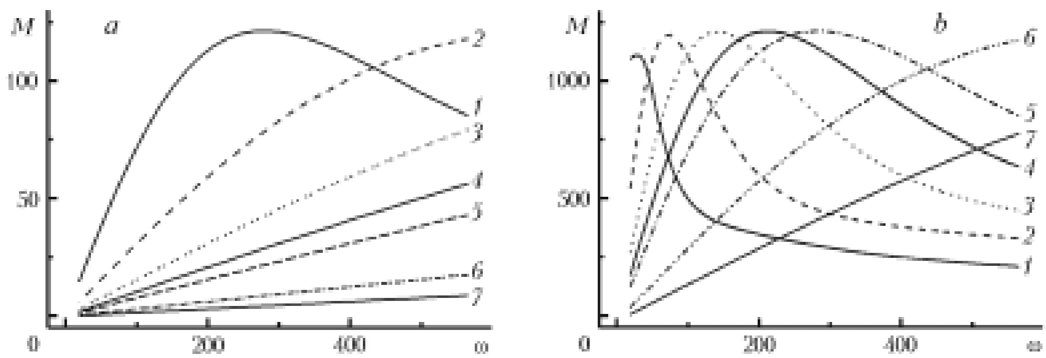

Fig. 3. Reduced moment of braking forces vs. angular rotational velocity of the part for different values of the resistance: 1) $\rho = 0.1 \cdot 10^{-7}$; 2) 0.25; 3) 0.5; 4) 0.75; 5) 1; 6) 2.7; 7) 5 $\Omega \cdot$m. $R = 10^{-2}$ m. $\mu = 1$ (a); $\mu = 10 - i0.1$ (b). M, N.

mum from being reached. For example, if the initial parameters of treatment (see Fig. 3) correspond to the characteristics to the left of the extremum, one should decrease the rotational frequency of the part in optimization, and if they correspond to the characteristics to the right of the extremum one should increase the degree of screening of the part by increasing the rotational frequency.

Thus, it has been shown that in magnetic abrasive treatment, in a number of cases the effective torque of electromagnetic-braking forces which is related to the action of eddy currents is a nonmonotonic function of the rotational frequency of a part. The position of the torque's maximum depends on the rotational frequency of the part, the form and value of the magnetic permeability, and the radius of the part. In the limit of low frequencies, the effective torque is determined by the expression characteristic of quasistatic magnetization. For materials possessing a low level of magnetism the transition limiting in frequency is asymptotically accurate. In the region of high frequencies and high conductivity of the material, the moment of braking forces corresponds to a decrease in viscous forces due to the electromagnetic screening of the part.

NOTATION

B, induction of the external magnetic field, T; E, axial component of the vector of the electric-field strength, V/m; H, azimuthal component of the external magnetic field, A/m; I_0 and I_1, Bessel functions of zero and first orders; k, wave vector, m^{-1}; M, reduced moment of braking forces, i.e., effective braking torque per unit length of the specimen (7), N; r, z, cylindrical coordinates; R, radius of the cylindrical part, m; V, volume of the part, m^3; W', power of the forces of dissipation of electromagnetic energy, W; ϑ, azimuthal coordinate, rad; μ_0, magnetic constant, H/m; μ, magnetic permeability of the material; μ_1 and μ_2, real and imaginary components of the magnetic permeability; ρ, specific electrical resistance of the material, $\Omega \cdot$m; σ, specific electrical conductivity of the material, $\Omega^{-1} \cdot m^{-1}$; ω, frequency of change of the magnetic field, rad/sec. Subscript: s, surface.

REFERENCES

1. S. V. Vonsovskii, *Magnetism* [in Russian], Nauka, Moscow (1971).
2. F. Yu. Sakulevich, *Principles of Magnetic Abrasive Treatment* [in Russian], Nauka i Tekhnika, Minsk (1981).
3. Yu. M. Baron, *Magnetic Abrasive Treatment of Products and Cutting Tools* [in Russian], Mashinostroenie, Leningrad (1986).
4. D. F. Ustinovich, *Study and Development of the Technological Process of High-Speed Magnetic Abrasive Treatment*, Author's Abstract of Candidate's Dissertation (in Engineering), Belorussian Polytechnic Institute, Minsk (1994).
5. V. P. Sobol' and O. N. Mazurenko, Influence of a Magnetic Field on Frequency Phenomena of Transfer in Conducting Media, *Izv. Belor. Inzh. Akad.*, No. 1, Pt. 3, 201–204 (1997).